

Spin excitonic and diffusive modes in superfluid Fermi liquids

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A role of a particle-particle p-wave spin interaction in Fermi liquids with s-wave pairing is studied. Depending on the sign of the interaction there arises either the new exciton collective mode below the pair-breaking threshold or the diffusive excitation mode above the threshold. The Landau parameters which control the interaction strength are evaluated for various systems: the dilute fermion gases, degenerate electron liquid, metals, atomic nuclei and neutron matter. The interaction removes also the square-root singularity in the phase space of pair breaking processes. It is shown how these effects influence the neutrino emissivity in the neutron Cooper-pair recombinations in neutron stars.

Processes with recombinations of Cooper pairs provide important information about interparticle interactions and the pairing mechanisms in different fermionic systems: ordinary superconductors [1], liquid ³He and ³He-⁴He mixtures [2], cold atomic gases [3], atomic nuclei [4], neutron stars [5, 6], and other systems. In superconductors, they are studied by absorption of infrared radiation or by the Raman scattering [7]. In the cold fermion atom gas one can use the Stokes scattering to detect the onset of the pairing [8]. Inverse pair breaking and formation (PBF) reactions constitute an important mechanism of the neutron star cooling [6, 9]. In these processes the energy is released in the form of neutrino-antineutrino pairs radiating off the star. Superburst ignition depth is sensitive to the value of the PBF emissivity in the inner neutron star crust [10]. In [11] the PBF processes are suggested to be responsible for the recently observed rapid cooling of the young neutron star in Cassiopeia A.

It was shown [12] that a residual interaction of single particle excitations, which does not contribute to pairing, can bind them in a state orthogonal to the Cooper pair, generating collective excitation modes in superconductors. For superfluid ³He the similar mechanism was studied in [13]. Interactions in the same spin channel, in which the pairing occurs, were studied so far. The influence of the interaction in one spin channel on the pairing in another channel has not yet been considered.

In this Letter we study the effects of the p-wave interaction in the spin-one channel on excitations in a Fermi system with the spin-zero pairing. We calculate response induced by the external spin- and helicity-density sources and show that depending on the sign of an effective interaction there appears either a new exciton mode or a diffusive excitation mode. Then we evaluate the strength of this effective interaction for different Fermi systems and, as an example, calculate the neutrino emissivity in the PBF processes for the neutron star with the neutron pairing in ¹S₀ state taking into account the effects of the new collective modes and correlations.

We use the Fermi-liquid theory approach extended to systems with pairing by Larkin and Migdal and by

Leggett [14]. For the processes induced by weak nucleon interactions this approach was adopted in Ref. [15]. Interactions in particle-particle (ξ) and particle-hole (ω) channels are essentially different. The interaction amplitude of two fermions with momenta $\vec{p} = p_F \vec{n}$ and $\vec{p}' = p_F \vec{n}'$ before and after the interaction in the ξ -channel is parameterized as $\hat{\Gamma}^\xi = \Gamma_0^\xi(\vec{n}, \vec{n}') (i\sigma_2') (i\sigma_2) + \Gamma_1^\xi(\vec{n}, \vec{n}') (i\sigma_2' \vec{\sigma}') (\vec{\sigma} i\sigma_2)$ and in the ω -channel as $\hat{\Gamma}^\omega = \Gamma_0^\omega(\vec{n}, \vec{n}') 1' 1 + \Gamma_1^\omega(\vec{n}, \vec{n}') \vec{\sigma}' \vec{\sigma}$. Here p_F stands for the Fermi momentum, \vec{n} and \vec{n}' are the unit vectors. The unit matrices 1 and 1' and the Pauli matrices $\vec{\sigma}$ and $\vec{\sigma}'$ act in the nucleon spin space. Superscript " ω " indicates that the amplitude in this channel is taken for $|\vec{q} \vec{v}_F| \ll \omega \ll \epsilon_F$, v_F is the Fermi velocity, ϵ_F is the Fermi energy and $q = (\omega, \vec{q})$ is the transferred 4-momentum. The coefficients of harmonic expansion of the scalar $\Gamma_0^{\xi, \omega}$ and spin $\Gamma_1^{\xi, \omega}$ amplitudes, the Landau parameters, should be either evaluated microscopically or extracted from analysis of the experimental data [4].

The singlet pairing in the Fermi liquid occurs owing to the attractive interaction, $a^2 \rho \Gamma_0^\xi = f_0^\xi < 0$. At zero temperature the pairing gap Δ follows from equation $-1/f_0^\xi = A_0/(a^2 \rho) = \ln(2\epsilon_F/\Delta)$, where a is the pole residue and ρ is the density of states at the Fermi surface. This expression is naturally generalized for finite temperature T , cf. Eq. (5) in the second paper of Ref. [15]. Since $g_0^\xi \equiv 0$ for the scattering of identical fermions, the spin interaction in the ξ -channel simplifies as $a^2 \rho \Gamma_1^\xi(\vec{n}, \vec{n}') = g_1^\xi(\vec{n} \vec{n}')$. It is usually assumed that the higher Legendre harmonics are much smaller [14]. Since we focus on the spin channel, the interaction Γ_0^ω decouples and can be dropped. In the ω -channel, $a^2 \rho \Gamma_1^\omega = g_0^\omega + g_1^\omega(\vec{n}' \vec{n})$. Contributions from the zeroth harmonics, g_0^ω , are accompanied by the factor v_F^2 , see Ref. [15], and for non-relativistic Fermi liquids under consideration (for $v_F^2 \ll 1$) can be put zero. Thus we remain with only tree relevant Landau parameters $f_0^\xi < 0$, g_1^ξ and g_1^ω . Let us first put g_1^ω zero and demonstrate the influence of the interaction in the ξ -spin-one channel, g_1^ξ , on the pairing effects in the ξ -spin-zero channel. Then we recover de-

pendence on g_1^ω .

Consider now an external perturbation, which couples to the spin density operator $\vec{s}(x) = \psi^\dagger(x)\vec{\sigma}\psi(x)$ and the helicity density operator $h(x) = \psi^\dagger(x)(\vec{\sigma}\vec{p} + \vec{p}\vec{\sigma})\psi(x)/(2m)$, where ψ is the spinor of a non-relativistic fermion, \vec{p} is the momentum operator, and m is the mass of the free fermion. From these quantities one can build the axial (A) fermion current $\mathcal{J}^\mu = (h, \vec{s})$. The Fourier transform of its bare components after the Fermi liquid renormalization becomes $J^{\omega,\mu}(\vec{n}, q) = (\vec{\sigma}\vec{\tau}_{A,1}^\omega, \vec{\sigma}\tau_{A,0}^\omega)$. Here $\tau_{A,0}^\omega = e_A/a$ and $\vec{\tau}_{A,1}^\omega = e_A v_F \vec{n}/a$ are ω -vertices, e_A is the effective charge of the quasiparticle. For $\Gamma_1^\omega = 0$, that we now exploit, the in-medium vertices are $\tau_{A,0}(\vec{n}, q) = \tau_{A,0}^\omega$ and $\vec{\tau}_{A,1}(\vec{n}, q) = \vec{\tau}_{A,1}^\omega(\vec{n}, q)$. For $\Gamma^\omega \neq 0$ these vertices are modified [15]. Additionally, in a system with pairing there arise new vertices responsible for the PBF processes: $\vec{\sigma}\tilde{\tau}_{A,0}(\vec{n}, q)$ and $\vec{\sigma}\tilde{\tau}_{A,1}(\vec{n}, q)$. They follow from the solution of the Larkin-Migdal equations [14, 15],

$$\tilde{\tau}_{A,0}(\vec{n}, q) = -\frac{g_1^\xi}{a^2 \rho} \left(\langle (\vec{n} \vec{n}') (N(\vec{n}', q) + A_0) \tilde{\tau}_{A,0}(\vec{n}', q) \rangle_{\vec{n}'} + \langle (\vec{n} \vec{n}') O(\vec{n}', q; -1) \tau_{A,0}^\omega \rangle_{\vec{n}'} \right), \quad (1a)$$

$$\tilde{\tau}_{A,1}(\vec{n}, q) = -\frac{g_1^\xi}{a^2 \rho} \left(\langle (\vec{n} \vec{n}') (N(\vec{n}', q) + A_0) \tilde{\tau}_{A,1}(\vec{n}', q) \rangle_{\vec{n}'} + \langle (\vec{n} \vec{n}') O(\vec{n}', q; +1) \vec{\tau}_{A,1}^\omega(\vec{n}') \rangle_{\vec{n}'} \right). \quad (1b)$$

The brackets indicate the angular averaging $\langle \dots \rangle_{\vec{n}} = \int \frac{d\Omega_{\vec{n}}}{4\pi} (\dots)$. The loop functions $O(\vec{n}, q; \pm 1) = \frac{1}{2} a^2 \rho (z_+ \pm z_-) g_T(\vec{n}, \omega, \vec{q})$, and $N(\vec{n}, q) = a^2 \rho z_+ z_- g_T(\vec{n}, \omega, \vec{q})$ with $z_\pm = (\omega \pm \vec{v} \vec{q})/(2\Delta)$, and the master function

$$g_T(\vec{n}, \omega, \vec{q}) = \Delta^2 \int_{-\infty}^{+\infty} \frac{d\xi_p}{\epsilon_+ \epsilon_-} \left[\frac{E_- F_-}{\omega^2 - E_-^2} - \frac{E_+ (1 - F_+)}{\omega^2 - E_+^2} \right],$$

where $E_\pm = \epsilon_+ \pm \epsilon_-$, $F_\pm = f(\epsilon_-) - f(\epsilon_+)$, $f(x) = 1/(\exp(x/T) + 1)$ and $\epsilon_\pm = [(\xi_p \pm \vec{v} \vec{q})^2 + \Delta^2]^{1/2}$. The solution of Eq. (1) is

$$\tilde{\tau}_{A,0} = -\frac{(\vec{v} \vec{q})}{2\Delta} \tau_{A,0}^\omega \gamma_\parallel^\xi \langle g_T(\vec{n}') (\vec{n}_q \vec{n}')^2 \rangle_{\vec{n}'}, \quad (2)$$

$$\tilde{\tau}_{A,1} = \frac{\omega}{q} \vec{n}_q \tilde{\tau}_{A,0} - \frac{\omega}{2\Delta} \gamma_\perp^\xi \langle g_T(\vec{n}') \frac{1}{2} [1 - (\vec{n}_q \vec{n}')^2] \rangle_{\vec{n}'} \vec{P}_\perp,$$

where $\vec{P}_\perp = \vec{n} - \vec{n}_q (\vec{n} \vec{n}_q)$, $\vec{n}_q = \vec{q}/|\vec{q}|$ and the correlation factors

$$\begin{aligned} [\gamma_\perp^\xi]^{-1} &= \frac{1}{3} C_0 + \langle \frac{\omega^2 - (\vec{v} \vec{q})^2}{4\Delta^2} g_T(\vec{n}) \frac{1}{2} [1 - (\vec{n} \vec{n}_q)^2] \rangle_{\vec{n}}, \\ [\gamma_\parallel^\xi]^{-1} &= \frac{1}{3} C_0 + \langle \frac{\omega^2 - (\vec{v} \vec{q})^2}{4\Delta^2} g_T(\vec{n}) (\vec{n} \vec{n}_q)^2 \rangle_{\vec{n}} \end{aligned} \quad (3)$$

are controlled by one effective interaction parameter

$$C_0 = 3/g_1^\xi - 1/f_0^\xi. \quad (4)$$

The singlet pairing occurs for $f_0^\xi < 0$ and $3f_0^\xi < g_1^\xi$. Then, if $g_1^\xi < 0$, we have $C_0 < 0$, otherwise the p-wave pairing is preferable. For $g_1^\xi > 0$ we have $C_0 > 0$.

The response of the Fermi system to the excitation (A) is determined by the symmetrical current-current correlator $\Pi^{\mu\nu}(q) = \frac{1}{2} \langle \text{Tr} \{ J^{\omega,\mu}(\vec{n}, q) J^\nu(\vec{n}, q) \} \rangle_{\vec{n}}$ with the in-medium current $J^\mu(\vec{n}, q) = (\vec{\sigma} \vec{\chi}_{A,1}(\vec{n}, q), \vec{\sigma} \chi_{A,0}(\vec{n}, q))$ expressed via the reduced current correlators derived in [15]:

$$\begin{aligned} \chi_{A,0}(\vec{n}, q) &= L(\vec{n}, q; -1) \tau_{A,0}(\vec{n}, q) + M(\vec{n}, q) \tilde{\tau}_{A,0}(\vec{n}, q), \\ \vec{\chi}_{A,1}(\vec{n}, q) &= L(\vec{n}, q; +1) \vec{\tau}_{A,1}(\vec{n}, q) + M(\vec{n}, q) \vec{\tilde{\tau}}_{A,1}(\vec{n}, q), \end{aligned}$$

where $M(\vec{n}, q) = -a^2 \rho z_+ g_T(\vec{n}, \omega, \vec{q})$, and $\frac{L(\vec{n}, q; \pm 1)}{a^2 \rho} = (\frac{z_\pm}{z_-} - 1) g_T(\vec{n}, (\vec{v} \vec{q}), \vec{q}) - (\frac{z_\pm}{z_-} - \frac{1 \mp 1}{2}) g_T(\vec{n}, \omega, \vec{q})$. The temporal and spatial components of the tensor are $\Pi^{00} = \langle \vec{\tau}_{A,1} \vec{\chi}_{A,1}(\vec{n}, q) \rangle_{\vec{n}}$, and $\Pi^{ij} = \delta^{ij} \langle \tau_{A,0} \chi_{A,0}(\vec{n}, q) \rangle_{\vec{n}}$ with

$$\begin{aligned} \frac{1}{3} \sum_i \Pi^{ii} &= e_A^2 \rho \left\langle \frac{(\vec{v} \vec{q})}{\omega - \vec{v} \vec{q}} [g_T(\vec{n}, (\vec{v} \vec{q}), \vec{q}) - g_T(\vec{n}, \omega, \vec{q})] \right\rangle_{\vec{n}} \\ &+ e_A^2 \rho \frac{v_F^2 \vec{q}^2}{4 \Delta^2} \gamma_\parallel^\xi \langle (\vec{n}_q \vec{n})^2 g_T(\vec{n}, \omega, \vec{q}) \rangle_{\vec{n}}, \\ \Pi^{00} &= v_F^2 \frac{1}{3} \sum_i \Pi^{ii} + e_A^2 \rho v_F^2 \langle g_T(\vec{n}, \omega, \vec{q}) \rangle_{\vec{n}} \\ &+ e_A^2 \rho v_F^2 \frac{\omega^2}{2 \Delta^2} \gamma_\perp^\xi \langle g_T(\vec{n}, \omega, \vec{q}) \frac{1}{2} [1 - (\vec{n}_q \vec{n})^2] \rangle_{\vec{n}} \\ &+ e_A^2 \rho v_F^2 \frac{\omega^2 - v_F^2 q^2}{4 \Delta^2} \gamma_\parallel^\xi \langle g_T(\vec{n}, \omega, \vec{q}) (\vec{n}_q \vec{n})^2 \rangle_{\vec{n}}. \end{aligned} \quad (5)$$

The mixed components are $\Pi^{i0} = \Pi^{0i} = \vec{n}_q^i \frac{\omega}{3|\vec{q}|} \sum_j \Pi^{jj}$.

From Eq. (2) we see that the external perturbation can induce a singular response in the PBF amplitudes at the values ω and \vec{q} corresponding to the poles of the functions γ_\perp^ξ and γ_\parallel^ξ . These poles determine the new transverse and longitudinal collective modes (spin excitons). For $\vec{q} = 0$, the longitudinal and transverse modes coincide and their frequency ω follows from the condition

$$C_0 + y^2 \Re \tilde{g}_T(y) = 0, \quad y = \omega/(2\Delta), \quad (6)$$

where $\tilde{g}_T(y) \equiv g_T(0, 2\Delta y - i0, 0)$.

Although the full inclusion of the g_1^ω -dependence is rather tedious, the modification of Eq. (6) is simply given by the replacement $\Re \tilde{g}_T(y) \rightarrow \Re \tilde{g}_T(y)/(1 + \frac{1}{3} g_1^\omega \Re \tilde{g}_T(y))$. For $|C_0| \gg 1$ it induces the shift

$$C_0 \rightarrow C = C_0/(1 + C_0 g_1^\omega/3). \quad (7)$$

This relation interpolates between the limits $|C_0| \ll 3/|g_1^\omega|$ when $C \approx C_0$, and $|C_0| \gg 3/|g_1^\omega|$ when $C \simeq (3/g_1^\omega)(1 - 3/(g_1^\omega C_0))$. So, parameter C controls effects of residual interactions on the PBF processes.

In the long wave-length limit (for $\omega > |\vec{q}|$) from Eq. (6) we get $\Im \Pi^{ij}(q) = \frac{\delta^{ij}}{3} \frac{\vec{q}^2}{\omega^2} \Im \Pi^{00}(\omega)$. The response function, having for $y \sim 1$ the form

$$\begin{aligned} R(y, C) &\equiv \frac{\Im \Pi^{00}}{e_A^2 \rho v_F^2} = \frac{C^2 \Im \tilde{g}_T(y)}{[C + y^2 \Re \tilde{g}_T(y)]^2 + [y^2 \Im \tilde{g}_T(y)]^2} \\ &+ \pi \frac{C^2}{y^2} \delta(C + y^2 \Re \tilde{g}_T(y)), \quad \Im \tilde{g}_T(y) = \frac{-\pi \tanh(\frac{y\Delta}{2T}) \theta(y)}{2y \sqrt{y^2 - 1}}, \end{aligned} \quad (8)$$

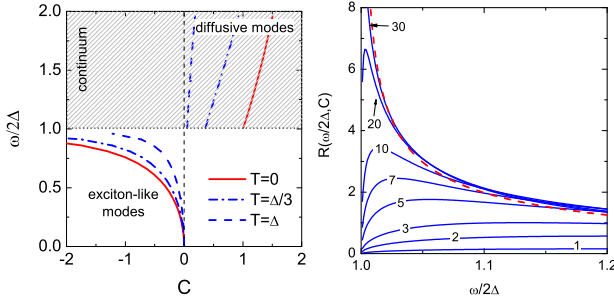


FIG. 1: Left: energies of the collective modes (for $\vec{q} = 0$) as functions of the parameter C for various values of Δ/T . Right: the response function $R(y)$ for $T = 0$ and $y > 1$ given by Eq. (8) for various values of the parameter C (solid lines). Dashed line shows the function $R(y, C \rightarrow \infty)$.

determines the probability of PBF processes. The cross section of the excitation scattering in matter is determined by this response function R .

Solutions of Eq. (6) are shown in Fig. 1 (left) as a function of C . For $C < 0$, solutions with $y < 1$ correspond to the undamped spin exciton branch at $\omega < 2\Delta$, since $\Im \tilde{g}_T(y < 1) = 0$. For $C > 0$, solutions exist only if $C > -\Re \tilde{g}_T(1 + 0)$ and $\omega > 2\Delta$. Since here $\Im \tilde{g}_T(y) \neq 0$, they constitute the diffusive spin mode. The frequency of the exciton mode increases with the increase of T and decreases for the diffusive mode. The response function $R(y)$ at $T = 0$ and $y > 1$ is plotted in Fig. 1 (right panel) by solid curves for various values of the parameter C . For $y > 1$ the function $R(y > 1, C)$ only weakly depends on the sign of the value C therefore we present it only for $C > 0$. The function $R(y, C \rightarrow \infty) = \Im \tilde{g}_T(y)$ is shown by the dashed curve. It has a square-root divergence at $y \rightarrow 1 + 0$, which is smeared out for finite values of C . Thus the finite value of C leads to a reduction of the spin response of the Fermi liquid close to the threshold for $\omega > 2\Delta$. A similar effect was discussed in Ref. [7] for the Raman scattering on metallic superconductors.

To estimate the value of our key parameter C we need to know Landau parameters g_1^ξ , f_0^ξ and g_1^ω . For a dilute Fermi gas we can use quasi-particle amplitudes derived in [16] up to the second order in the parameter $\zeta = 2a_{\text{eff}}p_F/\pi$, where a_{eff} is the effective scattering length. We derive $f_0^\xi = \zeta + \zeta^2(2\ln 2 + 1)/3$, $g_1^\xi = 3\zeta^2(1 - 2\ln 2)/5$, $g_1^\omega = -2\zeta^2(\ln 2 + 2)/5$ and obtain $C \approx -5.7/(a_{\text{eff}}p_F)^2$. For the neutron gas the value of the nn -scattering length is very large, $a \simeq 20$ fm, whereas the effective scattering length is much shorter [17], being determined, e.g., by the $V_{\text{low-}k}$ potential, $a_{\text{eff}} \simeq 2$ fm.

For more complex systems the parameters in the ξ -channel can be estimated with the help of the ω -Landau parameters in the s - p approximation of [18]

$$f_0^\xi = \sum_{l=0}^{\infty} (-1)^l \frac{A_l^s - 3A_l^a}{4}, \quad g_1^\xi = \sum_{l=0}^{\infty} (-1)^l \frac{A_l^s + A_l^a}{4}, \quad (9)$$

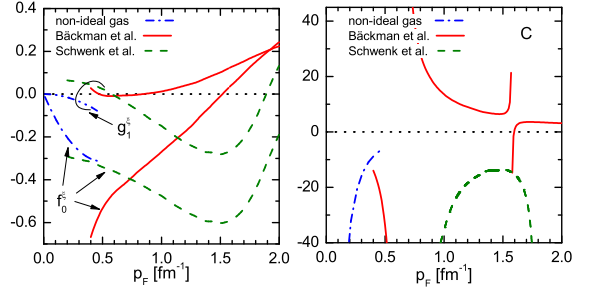


FIG. 2: Parameters f_0^ξ and g_1^ξ (left panel) and C (right panel) for the neutron matter reconstructed with the ω -Landau parameters calculated in Ref. [17, 21] using Eq. (9) as functions of the Fermi momentum.

here $A_l^s = f_l^\omega / (1 + \frac{f_l^\omega}{2l+1})$ and $A_l^a = g_l^\omega / (1 + \frac{g_l^\omega}{2l+1})$.

The Fermi-liquid approach was applied to the degenerate electron liquid in Ref. [19]. Using Table I and Table II of [19] we find $C = -2.54$, e.g., for a small value of the parameter $a_B p_F = 0.032$, where a_B is the Bohr radius.

For alkali metals at zero pressure the first three ω -harmonics are calculated in Ref. [20]. Applying (9) we then find for sodium $f_0^\xi(\text{Na}) = -0.11$, $g_1^\xi(\text{Na}) = -0.38$ and $g_1^\omega(\text{Na}) = -0.075$. Here the p-wave pairing is realized, since $C_0 > 0$, but the value $|C_0|$ is very small. Bearing in mind large uncertainties in estimates of the ω -Landau parameters one cannot exclude that $C < 0$ at $|C| \ll 1$. In the latter case we would deal with very pronounced effects of the spin exciton mode. This case can also be realized, if one allows a variation of the pressure. Thus presence or absence of the new exciton mode could tell about the kind of pairing in the given system. For potassium $f_0^\xi(\text{K}) = -0.56$, $g_1^\xi(\text{K}) = -0.89$ and, using $g_1^\omega(\text{K}) = -0.12$, we obtain $C = -1.48$.

For the nucleon matter several harmonics of the ω -Landau parameters were evaluated in many works, e.g., see Refs. [17, 21]. The parameter f_0^ξ related to the $1S_0$ pairing was also calculated, see [6]. Contrary, the g_1^ξ parameter is poorly known. Using results [17, 21] we reconstruct g_1^ξ and f_0^ξ with the help of Eqs. (9) and evaluate then parameters C_0 and C . For the neutron matter the results are shown in Fig. 2 in dependence of the Fermi momentum. We see that estimations of C are very uncertain due to discrepancy in different estimates of the ω -Landau parameters. Presented results show that might be $|C| < 10$ –20 at some densities in the range of the $1S_0$ pairing and even C might cross zero. Existence of regions where $C < 0$ implies a possibility to observe effects of the exciton modes.

Using the values of the ω -Landau parameters and their density dependence extracted from the atomic nuclear experiments [4, 22], we obtain $C \sim -10$ for $p_F \lesssim 1 \text{ fm}^{-1}$. Thus the exciton mode could manifest itself in the nuclear surface phenomena.

Now we apply Eq. (6) to calculate the neutrino emis-

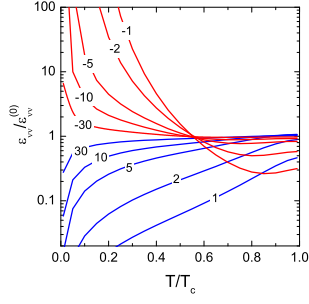


FIG. 3: The ratio $\varepsilon_{\nu\bar{\nu}}/\varepsilon_{\nu\bar{\nu}}^{(0)}$ as a function of T/T_c for various values of C (see curve labels).

sivity in the neutron star matter in the region of 1S_0 pairing. It is mainly determined by the neutron PBF process induced by the axial-vector current $\propto \mathcal{J}^\mu$ [15]; the vector current contribution is $O(v_F^4)$ and can be neglected [15, 23]. For one type of neutrino the emissivity then is given by [15]

$$\varepsilon_{\nu\bar{\nu}} = G^2 g_A^2 \int_0^\infty d\omega \omega \int_0^\infty d|\vec{q}| \frac{(q_\mu q_\nu - g_{\mu\nu}) \Im \Pi^{\mu\nu}(q)}{48 \pi^4 \exp(\omega/T) - 1},$$

where G and g_A are the weak-interaction and axial-vector coupling constants. The integration over $|\vec{q}|$ yields

$$\varepsilon_{\nu\bar{\nu}} \simeq \frac{8}{35 \pi^3} G^2 g_A^2 e_A^2 \rho v_F^2 \Delta^7 \int_1^\infty \frac{dy y^6 R(y, C)}{\exp(2y\Delta/T) + 1}, \quad (10)$$

where according to Eq. (8) there can be two contributions to $\varepsilon_{\nu\bar{\nu}}$: one, for arbitrary C , from the pair-breaking continuum with the diffusive modes at $\omega > 2\Delta$ and the other one, for negative C , from the spin-exciton mode with the frequency $\omega(\vec{q})$ at $0 < \omega(\vec{q}=0) < 2\Delta$. The later contribution is associated with the processes of breaking and formation of spin excitons. In the limit $|C| \rightarrow \infty$ the collective mode contribution vanishes as $\propto 1/|C|$ and we recover the result [15], $\varepsilon_{\nu\bar{\nu}}^{(0)}$, which follows from (10) after the replacement $R(y, C) \rightarrow R(y, C \rightarrow \infty) = \Im \hat{g}_T(y)$.

Effect of the finite value of C on the neutrino emissivity in the neutron PBF process is illustrated in Fig. 3, where we plot the ratio $\varepsilon_{\nu\bar{\nu}}/\varepsilon_{\nu\bar{\nu}}^{(0)}$ taking into account the standard temperature dependence of the 1S_0 pairing gap $\Delta(T) \simeq 3.1 T_c (1 - T/T_c)^{1/2}$ with T_c as the critical temperature. For $|C| \sim 5-10$, cf. Fig. 2 (right), the effect becomes pronounced for $T/T_c \lesssim 0.5$, yielding a suppression for $C > 0$ and an enhancement for $C < 0$. Thus in different density regions there may arise either an enhancement or a suppression of the PBF emissivity. Effect becomes even more pronounced for smaller values of $|C|$.

In conclusion, we found that the spin p-wave interaction in the particle-particle channel can produce new spin excitonic and diffusive modes in the Fermi system with the singlet pairing. This interaction leads also to smearing out of the threshold singularity in the Cooper-pair breaking reactions. We calculated the relevant coupling parameters for several Fermi systems. Spin excitons

may exist in superconducting potassium, in rare fermion gases, and in the neutron matter. In atomic nuclei the new spin exciton mode may manifest in the surface layer. Modification of the neutrino emissivity due to presence of spin excitonic and diffusive modes may have an impact on the neutron star cooling.

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